3. Optimal policy

As described in the last section, the optimal production and maintenance policy can be found by solving the following problem:

$$\text{Min} \qquad ECR(n,T,B,x_1,x_2,\cdots,x_n) \tag{5}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_i = T\,, \tag{6}$$

$$0 \le x_i \le T; \quad i=1,2,\cdots,n\,. \tag{7}$$

It is clear to see that the problem (5)-(7) is complicated and difficult or impossible to derive a closed form solution. Therefore, a search procedure is necessary for the optimal solution. To develop a solution procedure for the optimal policy, the Lagrange's method is used to solve the problem in (5)-(7).

First, we form the function

$$V(n,T,B,x_1,x_2,\cdots,x_n,\theta) = ECR(n,T,B,x_1,x_2,\cdots,x_n) + \theta\left(\sum_{i=1}^{n} x_i - T\right).$$

A candidate optimal solution can be found by solving the following system of equations:

$$\partial V(n,T,B,x_1,x_2,\cdots,x_n,\theta)/\partial T = 0\,, \tag{8}$$

$$\partial V(n,T,B,x_1,x_2,\cdots,x_n,\theta)/\partial x_i = 0\,, \quad i=1,2,\cdots,n\,, \tag{9}$$

$$\partial V(n,T,B,x_1,x_2,\cdots,x_n,\theta)/\partial B = 0\,, \tag{10}$$

$$\partial V(n,T,B,x_1,x_2,\cdots,x_n,\theta)/\partial \theta = 0\,. \tag{11}$$

The equations in (8)-(11) have the following form:

$$\frac{1}{T}\left\{\frac{(K+nv)D}{PT} + \frac{D}{PT}\sum_{i=1}^{n}\int_0^{x_i}[s\alpha P(x_i-y)+c_{i-1}(x_i-y)]f_i(y)\mathrm{d}y \times \frac{\{h[T(P-D)-B]^2+\pi B^2\}}{2T(P-D)}\right\} - \frac{h[T(P-D)-B]}{T} + \theta = 0\,, \tag{12}$$

$$\frac{D}{PT}\left\{d\int_0^{x_i}[s\alpha P + c'_{i+1}(x_i-y)]f_i(y)\mathrm{d}y + c_{i-1}(0)f_i(x_i)\right\} + \theta = 0\,, \quad i=1,2,\cdots,n\,, \tag{13}$$

$$\frac{\{-2h[T(P-D)-B]+2\pi B\}}{2T(P-D)} = 0\,, \tag{14}$$